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HIGHER ORDER NONLINEARITY IN ACCRETION DISKS: QPOS OF BLACK HOLE AND NEUTRON STAR SOURCES AND THEIR SPIN

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ABSTRACT

We propose a unified model to explain Quasi-Periodic-Oscillation (QPO), particularly of high frequency, observed from black hole and neutron star systems globally. We consider accreting systems to be damped harmonic oscillators exhibiting epicyclic oscillations with higher order nonlinear resonance to explain QPO. The resonance is expected to be driven by the disturbance from the compact object at its spin frequency. The model explains various properties parallelly for both types of the compact object. It describes QPOs successfully for ten different compact sources. Based on it, we predict the spin frequency of the neutron star Sco X-1 and specific angular momentum of black holes GRO J1655-40, XTE J1550-564, H1743-322, GRS 1915+105.

Subject headings: black hole physics — stars: neutron — gravitation — relativity

1. INTRODUCTION

The origin of Quasi-Periodic-Oscillation (QPO) and its properties are ill-understood phenomena in modern astrophysics. Observed QPO frequencies from black hole, neutron star, as well as white dwarf systems range from milli Hertz (mHz) to kilo Hertz (kHz) with varying properties. Among them most interesting and puzzling QPOs are the kHz ones. To date, more than 20 accreting neutron stars in Low Mass X-Ray Binaries (LMXB) have been found exhibiting such kHz frequency QPOs (van der Klis 2000; 2006). Several black holes have also been found exhibiting the high frequency (HF) QPOs of a few hundreds of Hz. The most puzzling cases are those when a pair of QPO forms, particularly in neutron star systems, and QPOs in the pair appear to be separated in frequency either by the order of the spin frequency of the neutron star, apparently for *slow rotators*, or by half of the spin frequency, for *fast rotators*. An example of the later cases is 4U 1636-53 with the spin frequency $\nu_s \sim 580$ Hz, while of the former ones is 4U 1702-429 with $\nu_s \sim 330$ Hz. Moreover, in several occasions the frequency separation decreases with the increase of one of the QPO frequencies. In black hole binaries also the HF QPOs have been found in pairs, but seem to appear at a $3:2$ ratio (Remillard et al. 2002) that favors the idea of a resonance mechanism behind their origin. The examples are GRO J1655-40, XTE J1550-564, GRS 1915+105 which exhibit the HF QPOs. The difference of the twin HF/kHz (van der Klis 2005) QPOs between black holes and neutron stars might be due to presence of a rotating magnetosphere in the later cases imprinting directly the spin frequency on the oscillations of the disk. However, sometimes QPOs from neutron stars, e.g. Sco X-1, also seem to appear at a $3:2$ ratio (Abramowicz et al. 2003).

Magnetically driven warping and precession instability are expected to occur in the inner region of an accretion disk (Lai 1999) by interaction between the surface current density in the disk and the stellar magnetic field. This produces motions at low frequencies, much lower than the orbital frequency accounting for the low frequency QPOs. This is similar to the Lense-Thirring precession shown by Stella & Vietri (1998). The mHz QPOs in some systems can be explained with magnetic warping process for strongly magnetized neutron stars (Shirakawa & Lai 2002). Titarchuk (2003), on the other hand, described QPO by the Rayleigh-Taylor instability associated with Rossby waves and rotational splitting. The effect of nonlinear coupling between g-mode oscillations in a thin relativistic disk and warp was examined by Kato (2004) for a static compact object.

A strong correlation between low and high frequency QPOs has been found by recent observations (Psaltis, Belloni & van der Klis 1999; Mauche 2002; Belloni, Psaltis & van der Klis 2002) of accretion disks around compact objects. Holding the relation over mHz to kHz range strongly supports the idea that QPOs are universal physical processes, independent of the nature of the compact object. Observing universal nature between the HF/kHz QPOs in black holes and neutron stars, McClintock & Remillard (2004) suggested that they are due to the orbital motion of the accreting matter near the marginally stable circular orbit. Titarchuk & Wood (2002) explained the correlation in terms of a centrifugal barrier model (see also Titarchuk, Lapidus & Muslimov 1998). Indeed, earlier by numerical simulation, Molteni, Sponholz & Chakrabarti (1996) showed QPO in the black hole accretion disks from the shock oscillation induced by the strong centrifugal barrier. Later, Chakrabarti & Manickam (2000) verified this analyzing observational data.

In early, it has been shown that QPOs may arise from nonlinear resonance phenomena in an accretion disk placed in strong gravity (e.g. Kluzniak et al. 2004; Petri 2005; Blaes et al. 2007). This is particularly suggestive because of their variation in frequency, while we know that the varying frequency in a characteristic range is a well studied feature of nonlinear oscillators. Indeed accretion dynamics and the disk structure are determined by the nonlinear hydrodynamic/magnetohydrodynamic equations with the effective gravitational potential which is not harmonic.

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Therefore, a nonlinear response is expected. The above authors argued for the resonance between epicyclic motions of accreting matter around a rotating compact object resulting QPOs, particularly the HF/kHz ones. However, the separation between vertical and radial epicyclic frequencies as a function of radial coordinate increases mostly with the increase of either of the frequencies. Although the separation of QPO frequencies in a pair observed from XTE J1807-294 appears constant (Linares et al. 2005) and from Aql X-1 seems increasing (Barret, Boutelier & Miller 2008) with the increase of either of the frequencies, in most of the cases it is opposite (modeled by Stella & Vietri 1999; Karas 1999) which contradicts the model based on the resonance between epicyclic motions. At radii smaller than the radius showing peak of radial epicyclic frequency, while the separation decreases with the increase of radial epicyclic frequency, the frequency separation is too large, much larger than the spin frequency of the star. Indeed, based on Cir X-1 data, Belloni, Méndez & Homan (2007) argued against the model. Therefore, one has to invoke new physics to describe multiple QPO features.

In the present paper, we propose a *global* resonance model based on higher order nonlinearity to describe black hole and neutron star QPOs together. The model successfully describes important features including variation of the QPO frequency separation in a pair as a function of QPO frequency itself. On the mission of reproducing observed QPOs, the model predicts the spin parameter (specific angular momentum or the Kerr parameter) of black holes as well as the spin frequency of a neutron star. The paper is organized as follows. In the next section, we establish the model describing the underlying physics briefly. In §3 and §4 we reproduce various properties of observed kHz/HF QPOs from six neutron stars and four black holes respectively, minimizing free parameters. Finally, we end with a summary in §5.

2. MODEL

Wagoner (1999) and Kato (2004) investigated small oscillations of geometrically thin accretion disks around a rotating compact object in the linear regime in the context of QPO. Later, Rezzolla et al. (2003) studied the radial oscillations of twodimensional accretion tori in the Schwarzschild spacetime. Kluzniak et al. (2004) investigated the response to a transient external perturbation of an ideal gas torus around a nonrotating black hole or neutron star. They found that the torus performs harmonic global oscillations in both the radial and the vertical directions with a variable amplitude in the later direction. These effects indicate the existence of a nonlinear coupling into the system.

We know that a system of N degrees of freedom has N linear natural frequencies denoting $\nu_1, \nu_2, \nu_3, ...\nu_N$ (Nayfeh & Mook 1979). Depending on the order of nonlinearity to the system, these frequencies are commensurable with certain relationships which may cause the corresponding modes to be strongly coupled and yield an internal resonance. If the system is excited by a mode of frequency ν_*, then frequencies might have the commensurable relationship exhibiting the resonance

$$a\nu_* = \sum_{i=1}^{N} b_i \nu_i, \tag{1}$$

(apart from all the primary and secondary resonance conditions $c\nu_* = d\nu_m$, where ν_m corresponds to the mth mode, a, b_i, c, d are integers) such that

$$a + \sum_{i=1}^{N} |b_i| = j, \tag{2}$$

where $j = k + 1$ when k is the order of nonlinearity. Certainly the order of nonlinearity determines the type of resonance conditions. For details, see Nayfeh & Mook (1979).

Now we consider an accretion disk to be a system of higher degree nonlinearity and described as a damped oscillator with higher degree anharmonicity. Then we consider the possible resonance therein. The resonance is expected to be driven by the combination of the strong disturbance by the compact object with spin ν_s and other already existent (weaker) disturbances in the disk particles (and hence disk) at the frequencies of the radial (ν_r) and vertical (ν_z) epicyclic oscillations given by (e.g. Shapiro & Teukolsky 1983)

$$\nu_r = \frac{\nu_o}{r}\sqrt{\Delta - 4(\sqrt{r} - a)^2}, \;\; \nu_z = \frac{\nu_o}{r}\sqrt{r^2 - 4a\sqrt{r} + 3a^2}, \tag{3}$$

where $\Delta = r^2 - 2r + a^2$, r is radial coordinate expressed in unit of GM/c^2, a is specific angular momentum (spin parameter) of the compact object expressed in unit of GM/c, G the Newton's gravitation constant, c is speed of light, M is mass of the compact object, and ν_o is orbital frequency of the disk particles given by

$$2\pi\nu_o = \Omega = \frac{1}{r^{3/2} + a}\,\frac{c^3}{GM}. \tag{4}$$

We describe the system schematically in Fig. 1. It can be seen to have composed of two oscillators, one in the radial direction (radial epicyclic motion) and other in the vertical direction (vertical epicyclic motion), with different spring constants. Note that spring constants here vary with radial coordinate. The basic idea is that the mode corresponding to ν_s arised due to the disturbance created by rotation of the compact object will couple to the ones corresponding to ν_r and ν_z, already exist in the disk, exciting new modes with frequencies $\nu_{r,z} \pm p\nu_s$ (Nayfeh & Mook 1979), where p is a number, e.g. $n/2$ when n is an integer ($1, 2$ for the present model). Now at a certain radius in the nonlinear regime where $\nu_s/2$ (or ν_s in the linear regime) is close to $\nu_z - \nu_r$ (Nayfeh & Mook 1979; Kluzniak et al. 2004), if $\nu_s/2$ (or ν_s) is also coincidentally close to the frequency difference of any two newly excited modes, then a resonance may occur which locks the frequency difference of two excited modes at $\nu_s/2$ (or ν_s).

For a neutron star having a magnetosphere coupled with the disk matter, the mode with ν_s can easily disturb disk particles through the interaction between surface current density in the disk and the stellar magnetic field and get coupled with the modes existing in the disk. Ghosh &

Lamb (1979a,b) showed, close to the stellar surface, how does the magnetic field channel matter to the polar caps. Later, Anzer & Börner (1980, 1983) showed the influence of the Kelvin-Helmholtz instability. However, LMXB have a very low surface field strength and a low critical fastness parameter compared to highly magnetized neutron stars (Li & Wickramasinghe 1997; Li & Wang 1999). Therefore, in this case, the strength of the disturbance is expected to be low and the accretion disk usually extends close to the stellar surface (Popham & Sunyaev 2001).

For a black hole, on the other hand, energy and angular momentum can be transferred with the magnetic field, associated with its disk, threading the horizon and connecting to the surrounding environment which may be looked as a variant of Blandford-Znajek process (Blandford & Znajek 1977). In fact, this kind of magnetic coupling process was already supported by XMM-Newton observations (e.g. Wilms 2001; Li 2002). This confirms the possible excitation of new modes in the disk around a spinning a black hole.

Note, as remarked by Karas (1999) in a similar context, that the disk particle size should be finite to visibly modulate the observed radiation due to the resonance for a long period. However, shearing effect destroys large particles. Abramowicz et al. (1992) suggested that formation of vortices in accretion disks could help in surviving finite size particles which was verified by Adams & Watkins (1995). While the origin of vortex is not fully understood yet, finite amplitude threedimensional secondary perturbations may form elliptical vortices in disks surviving for a long period (Mukhopadhyay, Afshordi, Narayan 2005; Mukhopadhyay 2006). In fact, Mukhopadhyay (2008), based on it, argued for the origin of turbulent viscosity in accretion disks.

Therefore, we rewrite (1) for an accretion disk as

$$(n-m+1)\nu_s = b_1\nu_r + b_2\nu_z \tag{5}$$

which leads to

$$\nu_r + \frac{n}{2}\nu_s = \nu_z - \frac{\nu_s}{2} + \frac{m}{2}\nu_s \tag{6}$$

with $-b_1 = b_2 = 2$ and m, n are integers. Now we propose the higher and lower QPO frequencies of a pair respectively to be

$$\nu_h = \nu_r + \frac{n}{2}\nu_s, \;\; \nu_l = \nu_z - \frac{\nu_s}{2}. \tag{7}$$

Hence, from (2) and (6) we understand the possible order of nonlinearity in accretion disks exhibiting QPOs $k = n - m + 4$. In the next section, we will reproduce several observational results, were not addressed by other QPO models, what justify the proposition.

In the disk around a neutron star, at an appropriate radius where $\nu_z - \nu_r \sim \nu_s/2$ (Kluzniak et al. 2004) and the resonance is supposed to take place, for $n = m = 1$, $\Delta\nu = \nu_h - \nu_l \sim \nu_s/2$, and for $n = m = 2$, $\Delta\nu \sim \nu_s$. $n = 1$ corresponds to a nonlinear coupling between the radial epicyclic mode and the disturbance due to spin of the neutron star, which results in $\Delta\nu$ locking in the nonlinear regime with $m = 1$. On the other hand, $n = 2$ corresponds to a linear coupling, which results in $\Delta\nu$ locking in the linear regime with $m = 2$. Hence, for the resonance n must be equal to m. The resonance with, e.g., $n = 2, m = 1$ for a neutron star corresponds to a linear coupling of the epicyclic and the disturbance modes. This results in the locking of $\Delta\nu$ ($\sim \nu_s/2$) to be in the nonlinear regime which is forbidden or very weak to observe. Similarly, $n = 3, m = 3$ may correspond to higher harmonics of the coupling in the nonlinear regime which are again expected to be weak to make any observable effects. It is expected that for a fast rotating neutron star the disturbance occurs in the nonlinear regime[2]. However, both ν_r and ν_z and then ν_h and ν_l vary as functions of radial coordinate. Therefore, $\Delta\nu = \nu_h - \nu_l$ varies as well. We argue that in a small range of radius, as given in Table 1, the resonance condition remains satisfied approximately, when $\Delta\nu$ decreases slowly with ν_l, hence explains observed data. Indeed observations show $\Delta\nu$ (may vary over 100Hz for a particular neutron star) to be of the order of ν or $\nu/2$ only, but not to be ν or $\nu/2$ exactly. In Fig. 2 we describe these facts for stars with $\Delta\nu \sim \nu_s/2$ and ν_s both. The vertical dot-dashed line indicates the location of the resonance.

For a black hole, however, in absence of a magnetosphere, the disturbance and then corresponding coupling between modes may not be nonlinear and occurs with the condition $\nu_z - \nu_r \lesssim \nu_s$, which results in the resonance locking at the linear regime with $n = m = 2$ which produces $\Delta\nu \lesssim \nu_s$ (sometimes $\sim 2\nu_s/3$) as shown in Figs. 3a,b. However, if we enforce the resonance to occur at marginally stable circular orbit, at the very inner edge of an accretion disk where the underlying physics is expected to be nonlinear than that in the relatively outer edge, then a nonlinear resonance occurs with $\Delta\nu \lesssim \nu_s/2$ (sometime $\sim \nu_s/5$) and $\nu_z - \nu_r \lesssim \nu_s$ for $n = m = 1$, as shown in Figs. 3c,d. In the following sections this is discussed in detail.

The difference between resonance conditions to form QPOs around black holes and neutron stars might be due to difference in their environment. For a neutron star, there is a rotating magnetosphere imprinting directly the spin frequency on the oscillations of the disk. However, energy and angular momentum of a black hole with magnetic field connecting to the surrounding disk can be transferred to the disk through the variant Blandford-Znajek process. This leads to the strong resonance, expected to be driven by the disturbance at the spin frequency of the black hole, even though the exact physical mechanism remains to be determined. Another important point to note is that for a black hole there is no strict definition of ν_s corresponding to its boundary layer related to the spin parameter a, while for a neutron star it is obvious due to presence of its hard surface. Here we assume that the effective boundary of a black hole is at marginally stable circular orbit r_{ms} and ν_s corresponds to the frequency of a test particle at r_{ms} [see equations (9), (10)].

From observational data we can determine the spin frequency of a neutron star. In computing QPO frequencies from our model, we need to determine the spin parameter a. If we consider a neutron star to be spherical in shape with the equatorial radius R, spin frequency ν_s, mass M, radius of gyration R_G, then moment of inertia and the

[2]Note that the strength of the disturbance is expected to be higher for a fast rotating neutron star compared to a slow rotating one and, hence, plausibly a nonlinear resonance is in the former case and a linear resonance in the later one. Indeed observations showing the apparent relation between QPO and spin frequencies of neutron stars support the argument.

spin parameter are computed

$$I = MR_G^2, \;\; a = \frac{I\Omega_s}{\frac{GM^2}{c}}, \tag{8}$$

where $\Omega_s = 2\pi\nu_s$. We know that for a solid sphere $R_G^2 = 2\,R^2/5$ and a for hollow sphere $R_G^2 = 2\,R^2/3$. However, the shape of a very fast rotating neutron star is expected to be deviated from spherical to ellipsoidal. Moreover, neutron stars are not expected to be perfect solid bodies. Hence, in our calculation we choose $0.35 \le (R_G/R)^2 \le 0.5$ in most cases (see e.g. Bejger & Haensel 2002; Cook, Shapiro & Teukolsky 1994).

On the other hand, for a black hole of mass M, a is the most natural quantity what we supply as an input. Corresponding angular frequency of a test particle at any radius r in the spacetime around it is then given by (e.g. Shapiro & Teukolsky 1983)

$$\Omega_{BH} = -\frac{g_{\phi t}}{g_{\phi\phi}} = \frac{2a}{r^3 + r\,a^2 + 2a^2}. \tag{9}$$

We now define

$$\Omega_s = 2\pi\nu_s = \Omega_{BH}(r = r_{ms})\,\frac{c^3}{GM}, \tag{10}$$

where r_{ms} is marginally stable circular orbit and light inside r_{ms} is practically not expected to reach us. Therefore, supplying a we can determine ν_s.

3. PROPERTIES OF NEUTRON STAR QPOS

For at least seven of neutron stars exhibiting twin kHz QPOs we know the spin frequency (e.g. Yin et al. 2007; Boutloukos & Lamb 2008) In most occasions, QPO frequencies vary with time and show that the frequency difference in the pair decreases with the increase of lower QPO frequency. Moreover, the frequency difference is of the order of half of the spin frequency of the star for fast rotators and of the spin frequency itself for slow rotators. We will show that our model successfully describes all the properties matching with observed data. In obtaining results, we choose primarily $M = 1.4M_\odot$, $(R_G/R)^2 = 0.4$ along with a guess radius R; all are free parameters. However, this choice does not suffice observation mostly. Hence, subsequently, we discuss results with other values of parameters.

3.1. *Fast rotators*

We analyze QPOs from three fast rotating neutron stars whose spin frequencies are known: KS 1731-260 (Smith, Morgan & Bradt 1997), 4U 1636-53 (Jonker, Mendez, & van der Klis 2002) and 4U 1608-52 (Mendez et al. 1998). For various sets of input values of M and R_G^2/R^2 we predict radius of the neutron star, given in Table 1, in reproducing observed data by our theory, depicted in Figs. 4a-c. KS 1731-260, till date, has shown to exhibit only one pair of QPO frequency established by our model easily as depicted in Fig. 4a. However, the resulting R is large/unrealistic if mass of the star is considered to be $M = 1.4M_\odot$. If we choose the star not to be a solid sphere or to be an ellipsoid with $(R_G/R)^2 = 0.5$ or 0.35, then the resulting radius decreases to $R = 16.4$km which is in accordance with realistic equations of state (EOS) [Friedman, Ipser & Parker 1986, hereafter eos1; Cook, Shapiro & Teukolsky 1994, hereafter eos2]. Figures 2a,b show the resonance to occur at $r_{QPO} = 8$ (indicated by the vertical dot-dashed line) for $n = m = 1$ with $\nu_z - \nu_r \sim \Delta\nu \sim \nu_s/2$.

For 4U 1636-53 and 4U 1608-52, on the other hand, twin peaks are observed in several occasions. For 4U 1636-53, our theory has an excellent agreement with observation with $\chi^2 \sim 28$, as shown in Fig. 4b. If we exclude only data point falling outside the theoretical prediction, which does not appear to follow the same trend as other points follow and hence the significance of its detection appears to be less, then $\chi^2 \sim 6$. If either $(R_G/R)^2$ or M of the star is considered to be deviated from 0.4 or $1.4M_\odot$ respectively, then our theory reproduces observed data in accordance with realistic EOS (eos1, eos2). While at smaller ν_l-s, results for 4U 1608-52 deviate from observed data, as shown in Fig. 4c, the predicted radii for all the parameter sets, except that with $R = 10$km, are well within the conventional range (eos1, eos2), given in Table 1. While the estimated χ^2 for 4U 1608-52 is large (> 100) for the entire set of observed data, it is only ~ 8 when we fit a part of it, discarding remaining data points from the computation as of 4U 1636-53, by the dot-dashed line shown in Fig. 4c. However, the important point to note is that the number of data points in either of the cases is poor which may question the reliability of χ^2 values. Similar resonance as of KS 1731-260 also occurs for these sources as well.

3.2. *Slow rotators*

Results for slow rotating neutron stars whose spin frequencies are known: 4U 1702-429 (Markwardt, Strohmayer & Swank 1999), 4U 1728-34 (van Straaten et al. 2002; Mendez & van der Klis 1999), are depicted in Figs. 4d,e. For the sets of input values of M and $(R_G/R)^2$ we predict various possible radii of the neutron star, given in Table 1. If we consider $M = 1.4M_\odot$, then the predicted R is too large with the choice of a spherical star. If the star is considered to be an ellipsoid and/or not to be a solid sphere and/or to have mass $M < 1.4M_\odot$, then R decreases to conventional values (eos1, eos2). On the other hand, all the parameter sets for 4U 1702-429, given in Table 1, correspond to high specific angular momentum of the neutron star. Therefore, we predict that the star 4U 1702-429 can not be a solid sphere and thus corresponding sets of parameters in rows one and three appear to be ruled out. Figure 4d shows a perfect agreement of our theory with observation with $\chi^2 < 1$. The spin parameter for cases of 4U 1728-34, given in Table 1, ranges $0.5 \lesssim a \lesssim 0.75$. The parameter set with $a \sim 0.75$ given in the corresponding row one appears to be ruled out when the star is not expected to be a solid sphere and the mass-radius combination does not follow any realistic EOS. While the estimated χ^2 for 4U 1728-34 is large (> 100) for the entire set of observed data, it is only ~ 7 when we fit a part of it, as of 4U 1608-52, by the dot-dashed line shown in Fig. 4e. However, as mentioned earlier, the poor number of data points may question the reliability of χ^2 values. In Figs. 2c,d we describe the resonance and corresponding formation of QPOs for 4U 1702-429. Clearly, the resonance occurs in $r_{QPO} = 10-11$, indicated by the vertical dot-dashed line, for $n = m = 2$ when $\nu_z - \nu_r \sim \Delta\nu/2 \sim \nu_s/2$. Similar resonance occurs for other slow rotators.

3.3. *Estimating the spin frequency of Sco X-1*

The spin frequency of Sco X-1 is not known yet. This source exhibits a pair of kHz QPO frequency with the separation varies in a range $\sim 225 - 310$ Hz (Mendez & van der Klis 2000). We show in Fig. 4f the observed variation of the frequency separation as a function of lower QPO frequency and compare it with that obtained from our model. Considering all possible sets of input parameters (along with ν_s which is now unknown) to obtain the best fit of observations, we present some of them in Table 1. We find that mass of Sco X-1 must be less than $1.4M_\odot$ and the sets of inputs with smaller ν_s and M fit the observed data better and argue that Sco X-1 is a slow rotator with $\nu_s \sim 280 - 300$. For the parameter set with $\nu_s = 292$, $\chi^2 \sim 66$ when the number of observed data points is 39. However, if we recompute it discarding data points falling far outside the theoretical prediction, as of 4U 1636-53, 4U 1608-52, 4U 1728-34, given by the dot-long-dashed curve, then $\chi^2 \sim 38$ with observed data points 35, assuring the excellence of the fitting.

3.4. *Quality factor*

Quality factor (in short Q-factor) of observed QPO frequencies has been found to be as large as 200. Any viable model for QPO should be able to reproduce this high Q-factor. Figure 5 describes Q-factor of the 775Hz QPO observed from 4U 1636-53, based on our model. The power depends mainly on mechanical resistance of the system along with epicyclic frequencies[3] and frequency of the neutron star. Following the principles of a forced vibrating system of coupled oscillators we obtain the power approximately shown in Fig. 5a. However, Fig. 5b shows that the power falls off rapidly away from the resonance radius, because the component of radial epicyclic frequency merges with the vertical epicyclic frequency with the increase of r. Therefore, the resonance condition remains satisfied roughly in a (narrow) range of radii, as given in Table 1. The change in amplitude of the power in that range of radii results in the observed variation of QPO frequency. Figure 5b shows that the QPO frequency of 4U 1636-53 varies in the range $r_{QPO} = 7.6 - 8.5$ (see Table 1) with amplitude of the power decreases upto one-third of peak amplitude. Similar features can also be found for other compact sources.

The value of mechanical resistance r_m of a system controls Q-factor. From the perturbed Navier-Stokes equation, it is easy to understand that r_m for an accretion disk, where QPOs occur, is proportional to the radial velocity gradient dv_r/dr of the background flow. While for a Keplerian accretion disk (Shakura & Sunyaev 1973) dv_r/dr is very small, for a sub-Keplerian advective disk (e.g. Mukhopadhyay 2003) it could be high which is difficult to perturb rendering high mechanical resistance. In Fig. 5c we show the variation of Q-factor with r_m. It clearly argues that a very large Q-factor is possible for QPOs from a Keplerian disk.

4. PROPERTIES OF BLACK HOLE QPOS

The pairs of observed HF QPO from black holes seem to appear at a 3 : 2 ratio. Mass (or possible mass range) of several such black holes, e.g. GRO J1655-40 (Orosz & Bailyn 1997; Shahbaz et al. 1999), XTE J1550-564 (Orosz et al. 2002), GRS 1915+105 (Greiner, Cuby & McCaughrean 2001), H1743-322 (Miller et al. 2006) is already estimated from observed data. However, the spin of them is still not well established. We estimate the spin parameter of black holes in describing their observed QPOs by our model given in Table 2.

We supply observed M (or range of M, whichever is available) and arbitrary values of a as inputs of our model to reproduce observed QPOs. Without knowledge of exact mass for GRO J1655-40, XTE J1550-564 and GRS 1915+105, the estimated range of mass from observed data is supplied for our computation and we find the corresponding suitable range of input parameter a reproducing observed QPOs. However, mass of H1743-322 appears to be less uncertain with an estimated value $\sim 11M_\odot$. So we use that value and find a suitable spin parameter. For clarity, we also choose a (hypothetical) range of mass around $\sim 11M_\odot$ for this black hole and check in what extent a could vary. Our theory is able to produce QPOs at a 3 : 2 ratio with their observed values for $n = m = 2$ at a radius outside r_{ms}. Figures 3a,b describe the resonance and corresponding formation of QPOs for the black hole GRO J1655-40 with $M = 6M_\odot$ and $\nu_s = 218.48$Hz. The vertical dot-dashed line indicates the location of the resonance. It clearly says that $\nu_z - \nu_r \lesssim \nu_s$ at $r_{QPO} = 4.93$ and the resonance occurs with $\Delta\nu \lesssim \nu_s$ (say, $\sim 2\nu_s/3$, as shown in Fig. 3b). Similar resonance happens for other black holes. The corresponding spin parameter ranges from ~ 0.6 to ~ 0.8, depending on mass of the black hole.

However, if we enforce QPOs to happen at r_{ms} strictly by changing mass (within observed range of estimate) and the spin parameter of a black hole, then they produce for $n = m = 1$ at a higher a. Figures 3c,d describe such a formation of QPOs for the source GRO J1655-40 with $M = 7.05M_\odot$ and $\nu_s = 802.48$Hz. It again implies that the resonance corresponds to $\nu_z - \nu_r \lesssim \nu_s$, but with $\Delta\nu \lesssim \nu_s/2$ (say, $\sim \nu_s/5$, as shown in Fig. 3d). However, in this case the required mass of black holes to reproduce observed QPOs may be out of the range estimated from observed data and the corresponding a appears to be similar/same for all four black holes. Interesting fact to note is that the either case favors the fourth order nonlinearity in accretion disks around black holes and none of the parameter sets predict an extremally rotating black hole.

In early, employing fully relativistic accretion disk model, the spin of black holes was estimated based on a spectral analysis of the X-ray continuum (Shafee et al. 2006; McClintock 2006). While their estimated spin for GRO J1655-40 is in accordance with ours, for GRS 1915+105 it is higher ($a > 0.98$). However, Middleton et al. (2006), based on a simple multicolor disk blackbody including full radiative transfer as well as relativistic effects, estimated spin of GRS 1915+105 to be ~ 0.7 which tallies with ours. Perhaps, different estimate is due to difference in method and the disk model used for computation. Indeed, the best model to describe a geometrically thick accretion disk is still a matter of controversy. In fact, Wang, Ye & Huang (2007) showed how

[3]Note that the spring constants of the underlying oscillators, which vary with radial coordinate, are related to the components of epicyclic frequencies.

does the spin estimate vary with different method.

5. SUMMARY

We have described QPOs, particularly the kHz/HF ones, observed from several neutron stars and black holes (altogether ten compact sources), by a single model and predicted their spin. The model has addressed the variation of QPO frequency separation in a pair as a function of the QPO frequency itself of neutron stars, observed in several occasions. We argue that QPO is a result of higher (fourth) order nonlinear resonance in accretion disks. The model predicts the most possible set of parameters: $R \sim 15 - 16$km, $M \sim 1 - 1.2 M_\odot$ for fast rotators; $R \sim 16 - 17$km, $M \sim 0.8 - 1 M_\odot$ for slow rotators, when $(R_G/R)^2 \sim 0.4$, that can explain all the observed QPO pairs. Discussing QPOs for five different neutron stars (KS 1731-260, 4U 1636-53, 4U 1608-52, 4U 1702-429, 4U 1728-34) whose spin frequencies are known, we have predicted the spin frequency of Sco X-1 which is unknown until this work. We argue that this is a slow rotator.

We have addressed QPOs from four black holes (GRO J1655-40, XTE J1550-564, H1743-322, GRS 1915+105) whose mass (or range of mass) has already been predicted from observed data and QPO pair seems to appear at a 3 : 2 ratio. Based on the present model, we have predicted their spin parameters (a-s) which are not well established yet. According to the present model, none of them is an extremally rotating black hole. As our model explains QPOs observed from several compact objects including their specific properties, of that in black holes and neutron stars both, it favors the idea of QPOs to originate from a unique mechanism, independent of the nature of compact objects. A successful model should be able to reproduce properties globally as the one does presented here.

Now the future job should be to perform numerical simulations to establish the resonance described here to confirm the model. Importantly, one should compare the energy released from the oscillation with that from rest of the disk (and the neutron star surface, if QPO is from a neutron star) to address visibility of oscillations. In this connection, the variation of the disturbance in the disk due to rotation of the compact object should be employed properly into the model to pinpoint the largest possible radius, beyond that QPO does not occur, accurately.

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Table 1
Physical parameters of neutron stars

fast rotator	ν_s	M	$(R_G/R)^2$	n, m	R	range of r_{QPO}
KS 1731-260	525.08	1.4	0.4	1	22.01	one pair at 6.6
KS 1731-260	525.08	1.1	0.5	1	16.4	one pair at 8
KS 1731-260	525.08	1.1	0.4	1	18.32	one pair at 8
KS 1731-260	525.08	0.9	0.35	1	16.4	one pair at 9.3
4U 1636-53	581.75	1.4	0.4	1	18.4	$6.6-7.4$
4U 1636-53	581.75	1.4	0.49	1	16.8	$6.7-7.5$
4U 1636-53	581.75	1.2	0.4	1	16.0	$7.5-8.4$
4U 1636-53	581.75	1.18	0.35	1	16.8	$7.6-8.5$
4U 1608-52	619	1.4	0.4	1	15.0	$7.0-8.9$
4U 1608-52	619	1.4	0.5	1	13.5	$7.0-8.9$
4U 1608-52	619	1.1	0.4	1	10.0	$8.5-10.7$
4U 1608-52	619	1.3	0.35	1	14.5	$7.4-9.4$
4U 1608-52	619	1.4	0.45	1	15.1	$6.9-8.9$
4U 1608-52	619	1.49	0.4	1	16.5	$6.6-8.5$
slow rotator	ν_s	M	$(R_G/R)^2$	n, m	R	range of r_{QPO}
4U 1702-429	330.55	1.4	0.4	2	28.0	$7.8-8.6$
4U 1702-429	330.55	1.0	0.5	2	18.8	$10.0-11.0$
4U 1702-429	330.55	1.1	0.4	2	23.0	$9.4-10.3$
4U 1702-429	330.55	0.83	0.35	2	18.5	$11.6-12.7$
4U 1728-34	364.23	1.4	0.4	2	22.5	$7.2-9.9$
4U 1728-34	364.23	1.2	0.5	2	16.0	$8.1-11.1$
4U 1728-34	364.23	1.1	0.4	2	16.5	$8.7-11.8$
4U 1728-34	364.23	1.1	0.35	2	17	$8.7-11.8$
4U 1728-34	364.23	1.0	0.4	2	17.5	$9.2-12.6$
	estimated ν_s					
Sco X-1	300.0	1.4	0.7	2	21.3	$7.4-8.8$
Sco X-1	540.0	1.4	0.6	1	15.5	$7.0-8.0$
Sco X-1	422.0	0.9	0.5	1	17.4	$9.9-11.5$
Sco X-1	540.0	1.2	0.4	1	16.0	$7.9-9.1$
Sco X-1	280.0	0.8	0.5	2	17.5	$11.4-13.3$
Sco X-1	292.0	0.81	0.35	2	18.6	$11.3-13.2$

Note.—ν_s is given in unit of Hz, M in $M_\odot$, R in km, radial coordinate where QPO occurs, r_{QPO}, in unit of GM/c^2.

Table 2
Physical parameters of black holes

black hole	M	estimated a	ν_h theory/observation	ν_l theory/observation	r_{QPO}	Δr	n, m
GRO J1655-40	$6-7$	$0.737-0.778$	450/450	300/300	$4.93-4.25$	$1.71-1.23$	2
GRO J1655-40	7.05	0.95	451.31/450	299.04/300	1.94	0	1
XTE J1550-564	$8-11$	$0.682-0.768$	276/276	184/184	$5.91-4.4$	$2.44-1.33$	2
XTE J1550-564	11.5	0.95	276.67/276	183.32/184	1.94	0	1
H1743-322	11.3	0.74	242.39/242	165.89/166	4.81	1.6	2
H1743-322	$8-15$	$0.644-0.82$	242/242	166/166	$6.5-3.5$	$2.86-0.7$	2
H1743-322	12.7	0.95	242.46/242	166.05/166	1.94	0	1
GRS 1915+105	$10-20$	$0.606-0.797$	168/168	113/113	$7.38-3.9$	$3.58-0.98$	2
GRS 1915+105	18.4	0.95	167.35/168	114.61/113	1.94	0	1

Note.—$\nu_{l,h}$ are given in unit of Hz, M in $M_\odot$, r_{QPO} and its distance from marginally stable orbit, Δr, are expressed in unit of GM/c^2.

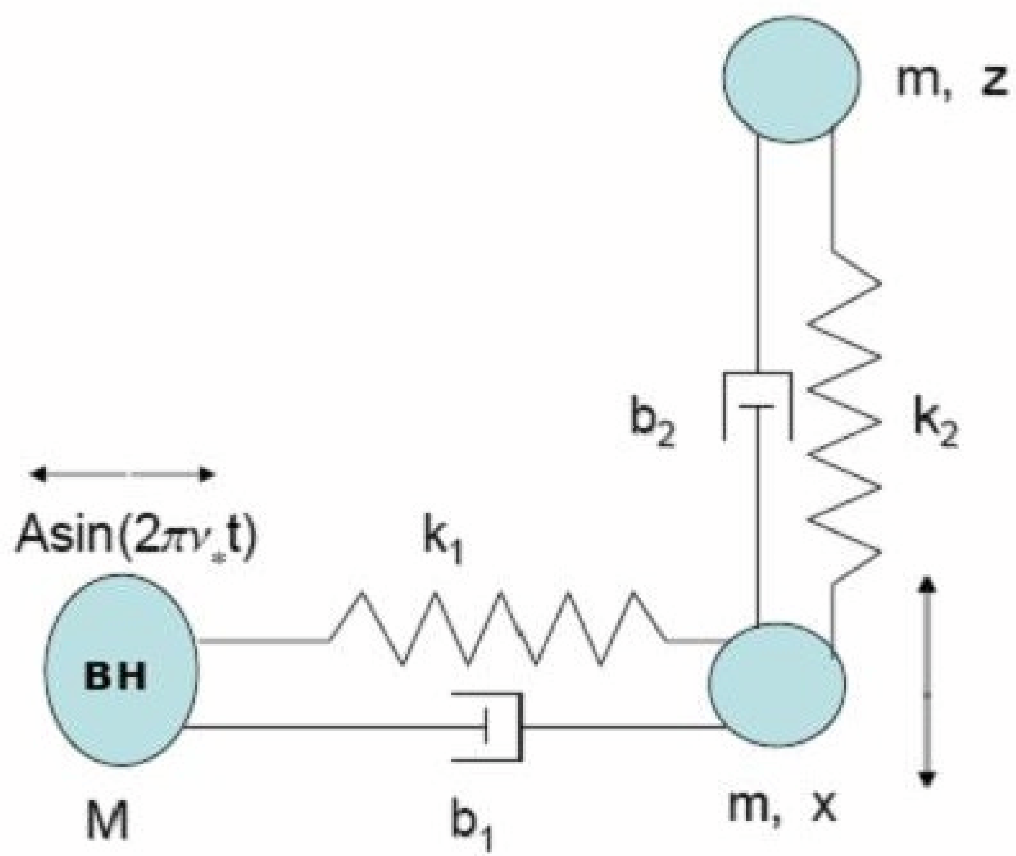

FIG. 1.— Cartoon diagram describing coupling of various modes in an accretion disk and the corresponding nonlinear oscillator. The oscillators describing by the spring constant k_1 and k_2 indicate respectively the coupling of spin frequency ν_* of the compact object with mass M to radial (x) epicyclic frequency and vertical (z) epicyclic frequency of a disk blob with mass m, where b_1 and b_2 represent corresponding damping factors respectively.

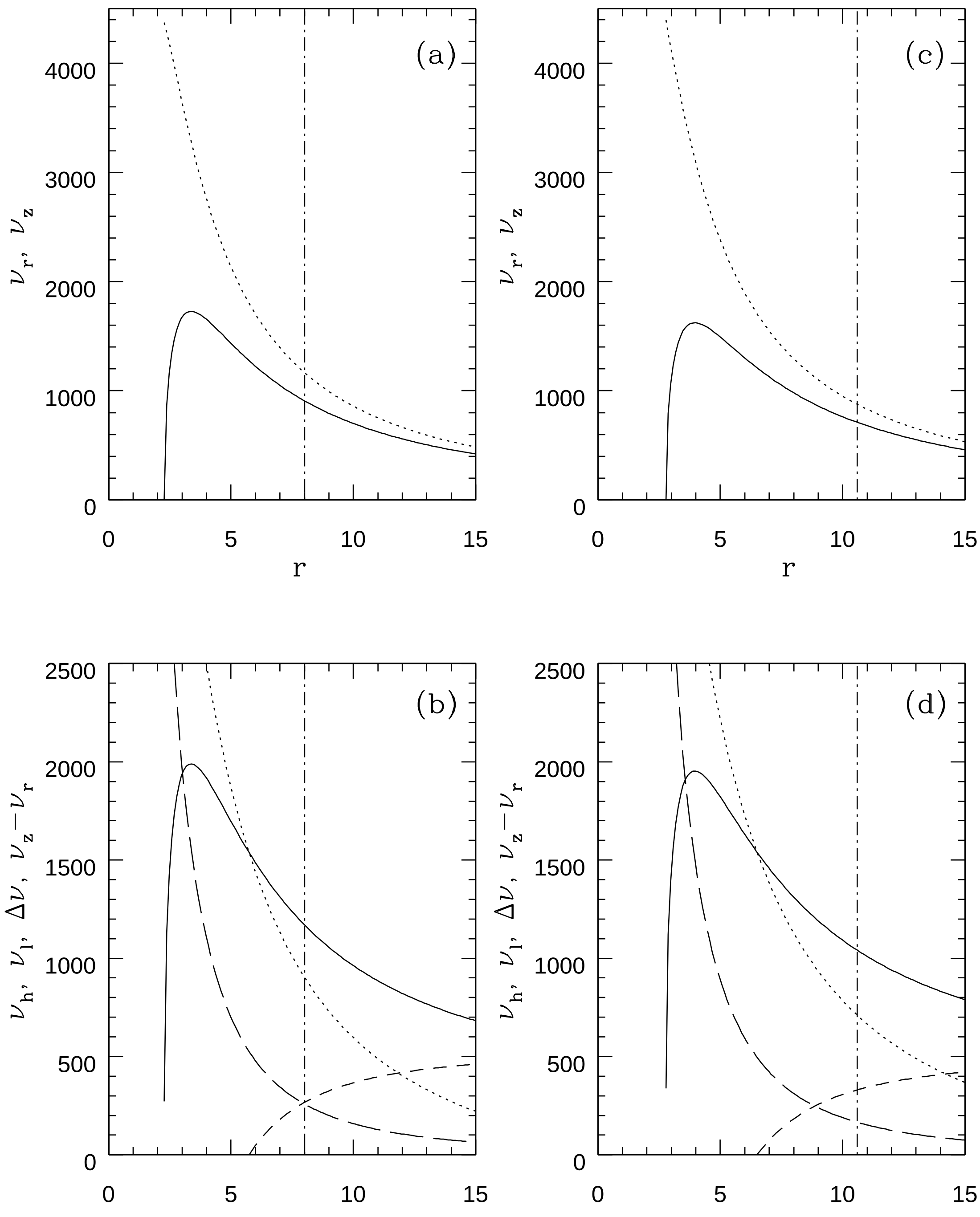

Fig. 2.— Variation of (a) radial (solid curve) and vertical (dotted curve) epicyclic frequencies in Hz as functions of radial coordinate in unit of GM/c^2, (b) higher (solid curve), lower (dotted curve) QPO frequencies, their difference (dashed curve), $\nu_z - \nu_r$ (long-dashed curve) in Hz as functions of radial coordinate in unit of GM/c^2, for the parameter set given in Table 1 for KS 1731-260 when $M = 1.1M_\odot$ and $R = 16.4$km. (c) Same as of (a), (d) same as of (b), but for 4U 1702-429 given in Table 1, when $M = M_\odot$ and $R = 18.8$km. The vertical dot-dashed curve indicates the location where the resonance occurs to generate QPO.

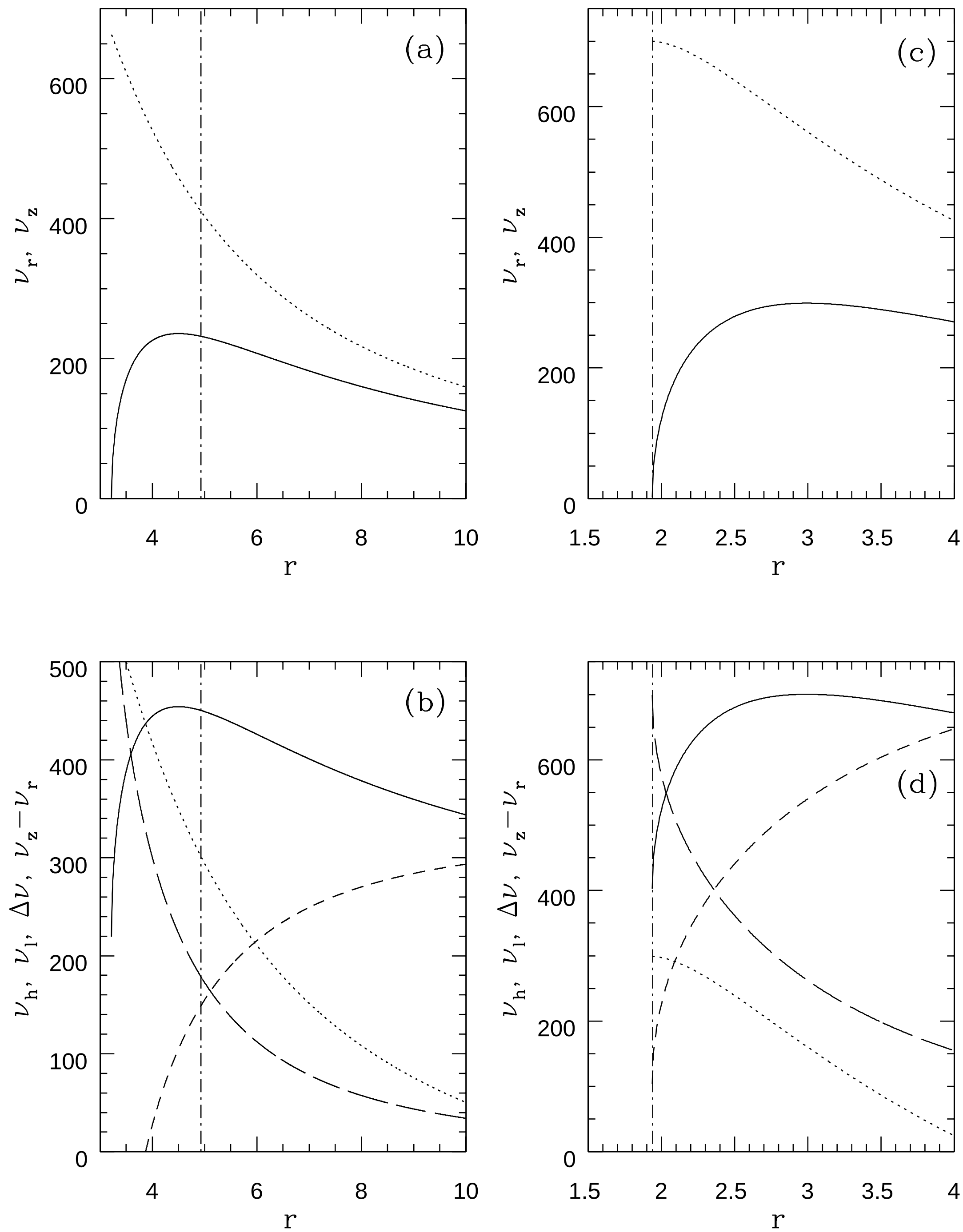

FIG. 3.— Variation of (a) radial (solid curve) and vertical (dotted curve) epicyclic frequencies in Hz as functions of radial coordinate in unit of GM/c^2, (b) higher (solid curve), lower (dotted curve) QPO frequencies, their difference (dashed curve), $\nu_z - \nu_r$ (long-dashed curve) in Hz as functions of radial coordinate in unit of GM/c^2, for the parameter set given in Table 2 for GRO J1655-40 when $M = 6M_\odot$, $n, m = 2$. (c) Same as of (a), (d) same as of (b), but for $M = 7.05M_\odot$, $n, m = 1$. The vertical dot-dashed curve indicates the location where the resonance occurs to generate QPO.

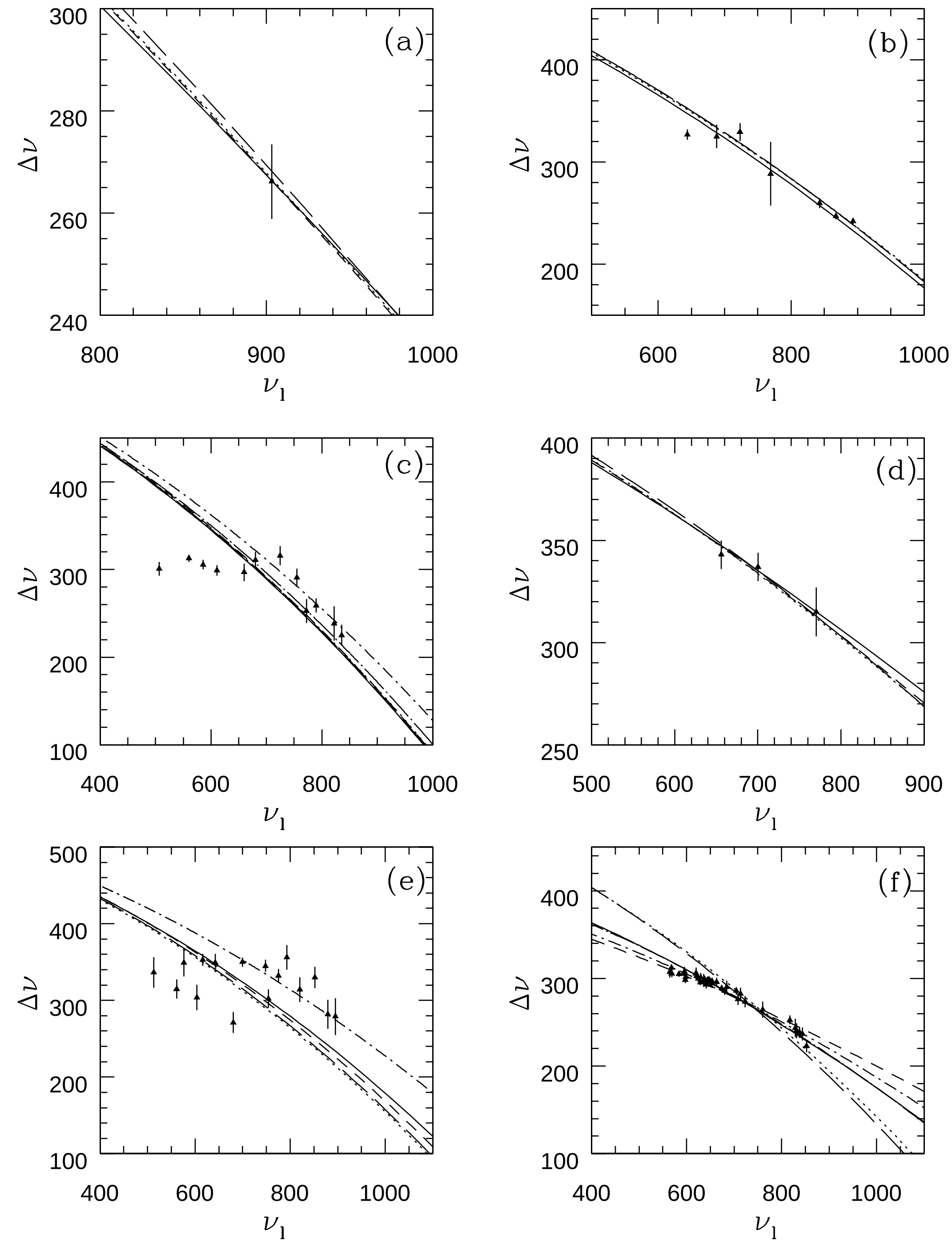

FIG. 4.— Variation of the QPO frequency difference in a pair as a function of lower QPO frequency for (a) KS 1731-260, (b) 4U 1636-53, (c) 4U 1608-52, (d) 4U 1702-429, (e) 4U 1728-34, (f) Sco X-1. Results for parameter sets given in Table 1 from top to bottom row for a particular neutron star correspond to the solid, dotted, dashed, long-dashed, dot-dashed (for 4U 1608-52, 4U 1728-34, Sco X-1 only), dot-long-dashed (for 4U 1608-52, Sco X-1 only) lines. The dot-dashed and dot-long-dashed lines in (c) and the dot-dashed line in (e) are to fit a part of the observed data points discarding the remaining ones. The triangles are observed data points along with error bars.

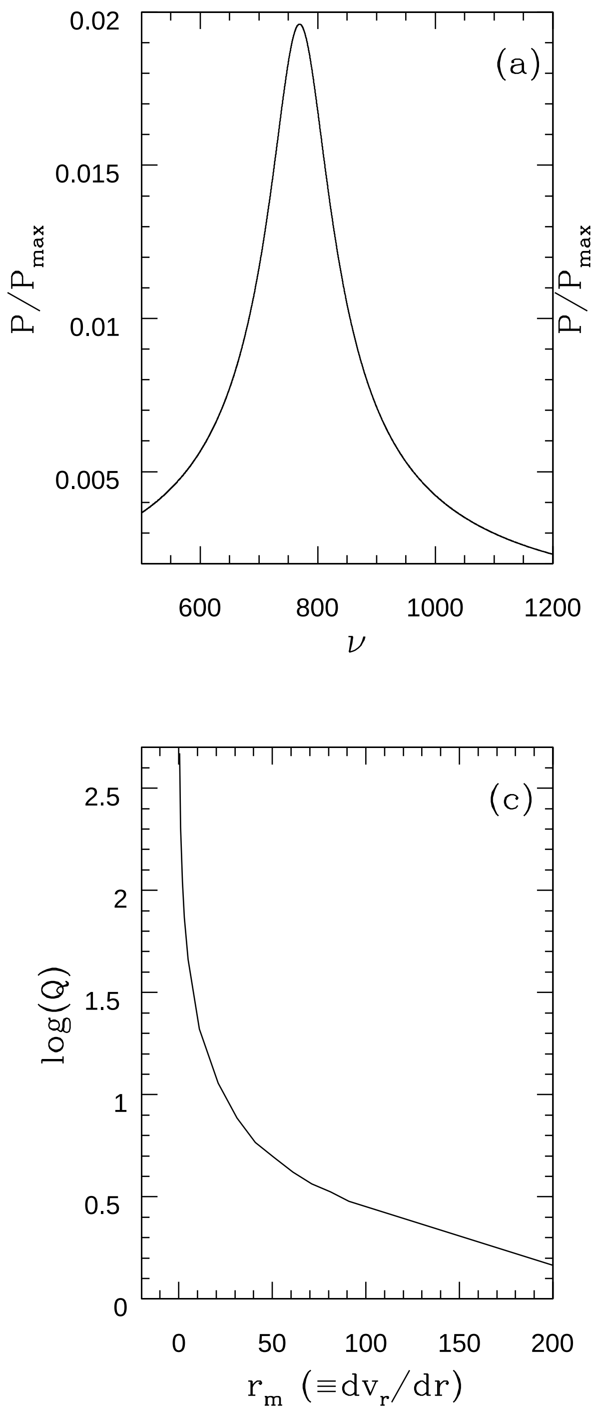

FIG. 5.— (a) Normalized Power-Spectrum for 4U 1636-53 showing 775Hz QPO, when $M = 1.18M_{\odot}$, $R = 16.8$km, $(R_G/R)^2 = 0.35$, mechanical resistance $r_m \equiv dv_r/dr \sim 50\text{sec}^{-1}$. (b) Variation of the normalized power in (a) as a function of radial coordinate in unit of GM/c^2. (c) Variation of Q-factor as a function of mechanical resistance of the flow in unit of sec^{-1}.